A. QUANTUM NATURE OF EMR

A1. Photon Theory (EMR)

Up to the end of the 19th century, EMR was viewed as a wave, which was confirmed by its many observed properties (e.g. refraction, diffraction, interference, and polarization). However, in the early 20th century, certain experiments dealing with light (as we will soon see) could only be explained by viewing EMR as composed of tiny particles of energy (photons). Scientists came to realize that EMR has both wave and particle properties, which is called wave-particle duality.

Energy of a Photon

According to photon theory, EMR is composed of bundles (quanta) of energy.

The energy of a photon (quantum of energy) is calculated as

\[ E_{\text{photon}} = hf \]

where \( f \) is the frequency of the EMR (in Hz)
\( h \) is Planck's constant \( (6.63 \times 10^{-34} \text{ J.s}) \)

Further, using the universal wave equation, \( v = f \lambda \)  or \( f = \frac{c}{\lambda} \)

\[ E_{\text{photon}} = \frac{hc}{\lambda} \]

where \( \lambda \) is the wavelength of the EMR (in m)

Note:
- High frequency EMR (such as UV, X-ray, gamma) has the highest energy photons - this is why they are more likely to do damage to human cells / DNA
- Since photon energies are so small, they are often measured in electron-Volts (eV)

\[ 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \]

Intensity (Brightness) of EMR

Intensity is the energy of EMR.

Classical (Wave) Theory:
Intensity = amplitude
Bright EMR = High A  
Dim EMR = Low A

Quantum Theory:
Intensity = Number of photons

\[ E_T = n \ E_{\text{photon}} \]

where \( n \) is the number of photons
Bright EMR = Large # of photons  
Dim EMR = Small # of photons
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Unit 5: Atomic Theory

HOMEWORK (Photon Theory)

A.  1. Convert:
   a) 273 GeV to Joules  
   b) 9.12 \times 10^{-15} \text{ J} to \text{ keV}

2. Compare how classical and photon theory would describe the following kinds of EMR:
   a) dim IR light  
   b) bright UV light

3. A certain blue light is created with a source vibration with a period of 1.44 \times 10^{-15} \text{ s}.
   What is the total energy of 500 photons (in eV)?

4. A 40.0 W monochromatic light source creates EMR with a frequency of 5.36 \times 10^{12} \text{ Hz}.
   a) What is the wavelength of this EMR (in \mu \text{m})?
   b) Determine the maximum number of photons emitted by this light source per second?

B.  5. Compare red light (700 nm) and violet light (400 nm) in terms of speed, frequency, and photon energy (in J). This is the range of visible light.

6. A certain type of EMR is shown by the graph. How many photons of this kind of EMR are required to create a total energy of 3.978 \times 10^{-15} \text{ J}?

7. A 120 W monochromatic bulb creates light with a wavelength of 5.60 \times 10^{-7} \text{ m}.
   a) What is the wavelength of this light in nanometres?
   b) What is the period of the source electrical vibration in the light bulb?
   c) If the bulb is 20.0% efficient, then determine the time required for this bulb to emit 8.00 \times 10^{22} \text{ photons}? Answer in minutes.

8. A Physics 30 conducts an experiment with a monochromatic bulb. He varies the frequency of the EMR and measures the number of photons emitted. All other variables are held constant. For this relationship, sketch the straightened curve, identify the equation of the straight line, determine the significance of the slope, and provide the units for the slope.

SOLUTIONS

1. a) 273 \times 10^9 \text{ eV} = 4.37 \times 10^8 \text{ J}  
   b) 57,000 \text{ eV} = 57.0 \text{ keV}

2. a) Classical: small amplitude, long \lambda  
   b) Classical: high amplitude, short \lambda  

3. f = 6.9444 \times 10^{14} \text{ Hz} ; E_{\text{photon}} = 2.875 \text{ eV} ; E_T = 1.44 \times 10^3 \text{ eV} = 1.44 \text{ keV}

4. a) \lambda = 5.597 \times 10^{-5} \text{ m} = 56.0 \mu \text{ m}
   b) Assume 100% efficiency: E_T = 40 \text{ J} ; E_{\text{photon}} = 3.5537 \times 10^{-21} \text{ J} ; n = 1.13 \times 10^{22} \text{ photons}

5. Red: Speed of light ; f = 4.29 \times 10^{14} \text{ Hz} ; E_{\text{photon}} = 2.84 \times 10^{-19} \text{ J} or 1.78 \text{ eV}
   Violet: Same speed; f = 7.50 \times 10^{14} \text{ Hz} ; E_{\text{photon}} = 4.97 \times 10^{-19} \text{ J} or 3.11 \text{ eV}

6. \lambda = 3.5 \text{ nm} ; E_{\text{photon}} = 5.6829 \times 10^{-17} \text{ J} ; n = 70 \text{ photons}

7. a) 560 \text{ nm} 
   b) f = 5.3571 \times 10^{14} \text{ Hz} ; T = 1.87 \times 10^{-15} \text{ s}
   c) E_{\text{photon}} = 3.5518 \times 10^{-19} \text{ J} ; E_T = 28,414 \text{ J} ; \Delta E = 142,070 \text{ J} ; t = 19.7 \text{ minutes}

8. Sketch n vs \frac{1}{f} ; Equation: n = \frac{\text{slope}}{f} ; \text{Slope} = \frac{P \cdot t}{h} ; \text{Units for slope: Hz or} \frac{W}{J}

p.97
A2. **Photoelectric Effect** (discovered by Heinrich Hertz, 1887)
- when high frequency EMR causes electrons to be emitted from a metal surface
  
  **Device**
  
  - when high frequency EMR is shone on the metal cathode, photoelectrons are emitted
  - they travel to the anode and form a photocurrent that is measured by the ammeter
  
  This is a called a **photocell**. It is a unique source of electrical energy, and it is the basic idea behind a solar panel.

**Classical Prediction**

Electrons will be emitted only when they absorb enough energy.

The higher the intensity of the incident EMR, the faster the energy is delivered to the electrons.

So, bright light would always emit electrons faster than dim light.

This would be true for any frequency of EMR.

**Actual Results** (defied classical theory)

- Only high frequency (short wavelength) EMR can emit photoelectrons
  - the frequency must be greater than a minimum frequency, called the **threshold frequency**

  If the incident frequency is too low \((f < f_0)\),
  - no electrons are emitted
  - no matter how bright the EMR is

  If the frequency is above the threshold frequency \((f > f_0)\),
  - electrons are emitted
  - no matter how dim the EMR is

- The brighter the EMR, the greater the photocurrent produced (assuming \(f > f_0\))

\[
\begin{array}{ll}
\text{bright light} &= \text{high } I \\
\text{dim light} &= \text{low } I
\end{array}
\]
A3. Einstein’s Explanation of Photoelectric Effect (verified wave-particle duality of EMR)

- Why must the incident EMR have a high frequency for electrons to be emitted?
  - the surface electrons of the metal require a certain amount of energy to be emitted
  - This energy required to dig the electron out of the metal is called the work function (W)
  - only one photon can be absorbed by an electron in the metal surface
  - based on the equation $E_{\text{photon}} = hf$, the incident photon will only have enough energy to emit
    electrons when the frequency is high enough

**If $f < f_0$:** The photon energy ($E = hf$) is smaller than the work function.
It does not have enough energy to remove the electron from the metal.
No current is created.

**If $f = f_0$:** The photon has just enough energy to remove the electron from the metal.
The photon energy ($E = hf_0$) is equal to the work function.

\[
W = \text{minimum photon energy to remove the electron}
\]
\[
W = hf_0
\]

**If $f > f_0$:** The photon energy is greater than the work function
The photon is fully absorbed by the surface electron in the metal
  - part of the photon's energy is used to dig the surface electron out of the metal
  - the rest of the photon's energy is in the form of kinetic energy

Conservation of Energy:
\[
E_{\text{Ti}} = E_{\text{Tf}}
\]
\[
E_{\text{photon}} = W + E_{k_{\text{max}}}
\]
where $E_{\text{photon}} = hf = \frac{hc}{\lambda}$

Note: The greater the frequency of the incident EMR,
  - the greater the energy of the incident photon
  - the greater the kinetic energy (speed) of photoelectron

- Why does bright EMR create more photocurrent?
  - the more intense the light,
    ⇒ the greater the number of photons in the EMR
    ⇒ each photon is absorbed by an electron in the metal
    ⇒ thus, more electrons will be emitted
    ⇒ more photocurrent will be created
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Unit 5: Atomic Theory

HOMEWORK (Photoelectric Effect I)

A. 1. The work function for cesium is 1.90 eV. What is the maximum wavelength that can emit electrons from the metal surface (in nm)? What kind of EMR is this (approximately)?

2. The minimum photon energy to emit electrons from a metal surface is $5.28 \times 10^{-19}$ J.
   a) What is the minimum frequency to emit electrons from this metal?
   b) If incident EMR with a frequency of $9.00 \times 10^{14}$ Hz is shone on this metal, then what is the maximum speed of the emitted photoelectrons?

3. Incident photons with 7.40 eV of energy are shone on a surface and the electrons are emitted with a maximum speed of $1.12 \times 10^6$ m/s. What is the work function of the metal (in J)?

Both A and B

4. The minimum frequency of light to emit electrons from a metal is yellow light. Describe the resulting photocurrent (no current, low current, or high current) for the following incident EMR:
   a) dim vs bright indigo light
   b) dim vs bright orange light

5. Describe and explain the effect on a simple photocell when the incident EMR experiences the following changes:
   a) increased frequency
   b) increased intensity

B. 6. When incident photons are shone on a metal surface, electrons are emitted with a maximum kinetic energy of 2.60 eV. If the minimum frequency to emit electrons from this metal is $8.94 \times 10^{14}$ Hz, then what is the wavelength of the incident EMR (in nm)? Type of EMR?

7. The maximum wavelength to emit photons from a metal surface is 428 nm. When a certain type of incident EMR is shone on the metal, the electrons are emitted with a maximum speed of $8.59 \times 10^5$ m/s. What is the frequency of the incident EMR?

8. When EMR is shone on a metal surface, it creates a photocurrent of 58.0 mA. What is the minimum number of photons hitting the metal surface in 7.00 seconds?Assumptions?

9. White light is shone on a metal surface with a work function of 1.94 eV. What is the greatest possible speed of the emitted electrons from the metal?

SOLUTIONS

1. $f_0 = 4.5894 \times 10^{14}$ Hz; $\lambda = 6.54 \times 10^{-7}$ m = 654 nm. This is orange / yellow light.

2. a) $W = 5.28 \times 10^{-19}$ J; $f = 7.96 \times 10^{14}$ Hz
   b) $E_{k\ max} = 6.87 \times 10^{-20}$ J; $v_{max} = 3.88 \times 10^5$ m/s

3. $E_{\ photon} = hf = 1.184 \times 10^{18}$ J; $E_{k\ max} = 5.7138 \times 10^{-19}$ J; $W = 6.13 \times 10^{-19}$ J

4. a) $f > f_0$, so electrons are emitted. Dim indigo = low current, Bright indigo = high current
   b) $f < f_0$, so no electrons are emitted by any type of orange light

5. Higher $f$ means higher photon energies, so e- are emitted at higher speeds. Higher intensity means more photons, and thus, more photocurrent (assuming $f > f_0$).

6. $E_{k\ max} = 4.16 \times 10^{-19}$ J; $W = 5.9272 \times 10^{-19}$ J; $\lambda = 1.97 \times 10^{-7}$ m = 197 nm (UV)

7. $W = 4.6472 \times 10^{-19}$ J; $E_{k\ max} = 3.361 \times 10^{-19}$ J; $f = 1.21 \times 10^{15}$ Hz

8. $q = 0.406$ C; $n = 2.54 \times 10^{18}$ photons (Each photon emits 1 e-; all e- reach the other plate)

9. Shorter wavelengths have more photon energies. So, choose 400 nm.
   $W = 3.104 \times 10^{-19}$ J; $f = 7.50 \times 10^{14}$ Hz; $E_{k\ max} = 1.8685 \times 10^{-19}$ J; $v_{max} = 6.40 \times 10^5$ m/s

p.100
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**A4. Millikan’s Verification of Einstein**

- Millikan verified Einstein’s Photoelectric equation using the device shown.

The back voltage acts to slow down the photoelectrons (decrease their $E_k$).

As the back voltage increases, the deeper electrons do not have enough $E_k$ to reach the other plate (but the surface electrons still do). So, current decreases.

If the back voltage increases until the current just becomes zero (called $V_{stop}$), even the surface electrons have lost all their $E_k$ by the time they reach the other plate.

**Equation:**

$V_{stop}$ is sufficient (barely) to reduce the surface photoelectron’s max kinetic energy to zero.

\[
\Delta E_k = -\Delta E \\
E_{k_f} - E_{k_i} = -q \Delta V \\
0 - E_{k_{\text{max}}} = -q V_{stop}
\]

So,

\[
E_{k_{\text{max}}} = q V_{stop}
\]

where

- $E_{k_{\text{max}}}$ is the max kinetic energy of the surface electrons (in J)
- $q$ is the magnitude of the charge of the photoelectron (in C)
- $V_{stop}$ is the minimum back voltage required to stop the photocurrent (in V)

$V_{stop}$ gave Millikan a way to measure the $E_{k_{\text{max}}}$ of the surface photoelectrons

**Millikan’s Experiment**

Millikan discovered that there was a linear relationship between $E_{k_{\text{max}}}$ and $f$

\[
y = m x + b
\]

$E_{k_{\text{max}}} = h f - W$

This verified Einstein’s photoelectric equation:

\[
y = m x + b
\]

or

\[
h f = W + E_{k_{\text{max}}}
\]

$E_{\text{photon}} = W + E_{k_{\text{max}}}$
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**Unit 5: Atomic Theory**

**HOMEWORK** (Photoelectric Effect II)

A. 1. The minimum potential difference to stop photocurrent is 3.60 V. What is the maximum speed of the photoelectrons as they leave the cathode?

2. The minimum energy to remove electrons from a metal surface is 2.08 eV. If the incident EMR shone on the metal has a frequency of $1.67 \times 10^{15}$ Hz, then what is the minimum back voltage to prevent photocurrent?

**Both A and B**

3. A Physics 30 student investigated the relationship between incident frequency and the resulting $E_{k\text{max}}$ of the photoelectrons. The results are shown in the graph.

Using only the line of best-fit, determine:

   a) the threshold frequency
   b) Planck’s constant
   c) the work function

B. 4. When a certain type of EMR is shone on a metal, the cut-off (stopping) potential difference for a metal is 2.75 V. If the minimum frequency to emit photoelectrons from this metal is $4.35 \times 10^{14}$ Hz, then what is the wavelength of the incident EMR (in nm).

5. The maximum wavelength to emit photoelectrons from a metal is 565 nm. If the incident photons have 6.70 eV of energy, then what is the minimum back voltage to stop the photocurrent?

**SOLUTIONS**

1. $E_{k\text{max}} = 5.76 \times 10^{-19}$ J ; $v_{\text{max}} = 1.12 \times 10^6$ m/s

2. $W = 3.328 \times 10^{-19}$ J ; $E_{k\text{max}} = 7.7441 \times 10^{-19}$ J ; $V_{\text{stop}} = 4.84$ V

3. a) $f_0 = x\text{-int} \approx 5.8 \times 10^{14}$ Hz
   b) $h = \text{slope} \approx 6.7 \times 10^{-34}$ Js
   c) $W = hf_0 \approx 3.9 \times 10^{-19}$ J

4. $E_{k\text{max}} = 4.40 \times 10^{-19}$ J ; $W = 2.8841 \times 10^{-19}$ J ; $\lambda = 2.73 \times 10^{-7}$ m = 273 nm

5. $W = 3.502 \times 10^{-19}$ J ; $E = hf = 1.072 \times 10^{-18}$ J ; $E_{k\text{max}} = 7.1996 \times 10^{-19}$ J ; $V_{\text{stop}} = 4.50$ V
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Unit 5: Atomic Theory

A5. Compton Effect (1923)

**Photon Momentum**
- according to classical theory, photons are pure energy and have no mass
- thus, they should have no momentum \( p = mv = 0 \)
- but, Compton suggested that photons could have **equivalent mass**
  That is, they can behave as if they do have mass and collide with other particles.
- he calculated the momentum of a photon in the following way:

\[
\text{Since } m = \frac{E}{c^2} \text{ and } \nu = c \text{, it follows that } p = mv = \left( \frac{E}{c^2} \right) c = \frac{E}{c} \]

\[
E = pc
\]

where
- \( E \) is the photon energy (in Joules)
- \( p \) is the photon momentum (in kg-m/s)
- \( c \) is the speed of light in air / vacuum

Based on this equation, high energy (i.e. high frequency, short wavelength) photons have the most momentum
- e.g. X-rays and gamma rays

**Compton Effect**
- provided evidence for photon momentum (irrefutable evidence that EMR behaves as a particle)

**Experiment:**
- directed high frequency (short \( \lambda \)) X-rays through thin carbon foil

\[\text{\begin{center}
\begin{tikzpicture}
\node at (0,0) {C foil};
\node at (2,2) {e- emitted};
\node at (-1,0) {short \( \lambda \) x-rays};
\node at (2,-2) {longer \( \lambda \) x-rays deflected};
\draw[->,thick] (0,0) -- (2,2);
\draw[->,thick] (0,0) -- (-1,0);
\draw[->,thick] (0,0) -- (2,-2);
\end{tikzpicture}
\end{center}}\]

**Results:**
- an electron was emitted (photoelectric effect)
- most X-rays were deflected and they emerged at a longer \( \lambda \)
  (i.e. at a lower frequency, lower energy)

\{ This is called the **Compton Effect** \}

p.103
The results of the Compton experiment defied classical theory:
- there should be no deflection
- frequency (and thus wavelength, if the medium does not change) should never change once
  the wave leaves the source

**Compton’s explanation**

He showed that the x-ray photon was having an **oblique collision** with an electron in orbit around
a carbon atom. In this way, the photon was acting like a particle. This is illustrated below:

![Diagram of Compton scattering](image)

Compton applied the laws of conservation of energy and momentum to this collision.

**Conservation of energy**

\[
E_{T_i} = E_{T_f} \\
E_{x-ray} = E_{k(e^-)} + E'_{x-ray}
\]

**Conservation of momentum**

\[
\vec{p}_T = \vec{p}'_T \\
\vec{p}_{x-ray} = \vec{p}_{e^-} + \vec{p}'_{x-ray}
\]

This would be a 2-D vector analysis.

Compton combined these two laws and derived a formula to predict the change in wavelength of
the x-ray photon:

\[
\Delta \lambda = \frac{h}{mc} (1 - \cos \theta)
\]

where
- $\Delta \lambda = \lambda' - \lambda$ is the change in wavelength of the x-ray photon (in m)
- $m$ is the mass of the particle (usually an electron) hit by the photon (in kg)
- $h$ is Planck’s constant (use $6.63 \times 10^{-34}$ J-s)
- $c$ is the speed of light
- $\theta$ is the angle of deflection of the x-ray photon (w.r.t. its initial direction)

**Quantum nature of EMR is conclusive**

Because the Compton effect could not (in any way!) be explained by classical theory, most
scientists became convinced of the particle nature of EMR.

p.104
For the questions below, ignore relativistic effects.

A. 2. A photon has 450 MeV of energy.
   a) What is its wavelength?  
   b) What is its momentum?

3. What wavelength of photon would have the same momentum as an alpha particle travelling at $5.20 \times 10^7$ m/s?

4. Sketch the vector triangle that illustrates conservation of momentum.
   An x-ray photon, moving towards the South, collides with a stationary electron.
   After the collision, the electron is travelling East.

5. A 2.80 pm x-ray photon deflects off a proton (initially at rest). If the proton has a final kinetic energy of $4.33 \times 10^{-14}$ J, then determine the wavelength (in pm) of the deflected x-ray photon.

6. A $4.00 \times 10^{-15}$ m (i.e. 4.00 fm) x-ray photon is moving East and collides with a stationary proton. After the collision, the x-ray photon is travelling due South. Determine:
   a) the frequency of the x-ray photon after the collision
   b) the kinetic energy of the electron after the collision
   c) the proton’s angle of deflection and final momentum

B. 7. A photon has a momentum of $5.07 \times 10^{-22}$ kg-m/s.
   a) What is its frequency?  
   b) How much energy does it have (in keV)?

8. How fast must a proton be travelling to have the same momentum as a 700 keV photon?

9. When an x-ray photon deflected off an electron, its wavelength increased by 1.70 pm. Determine the x-ray photon’s angle of deflection (w.r.t. its initial direction).

10. Sketch the vector triangle that illustrates conservation of momentum:
     A photon, travelling West, collides with a stationary electron. After the collision, the electron is moving at $20^\circ$ N of W and the photon is moving at $50^\circ$ S of W.

11. A 2.10 pm x-ray photon, moving East, collides with a stationary electron. After the collision, the x-ray photon is moving at $75.0^\circ$ N of E. Determine:
    a) the wavelength (in pm) of the x-ray photon after the collision
    b) the kinetic energy of the electron after the collision
    c) the electron’s angle of deflection
12. Compare and contrast the Compton effect with the photoelectric effect.

**SOLUTIONS**

1. $\theta = 110^\circ$ ; $\Delta \lambda = 3.26$ pm ; $\lambda_f = 9.06$ pm
2. $E = 7.2 \times 10^{-11}$ J  
   a) $\lambda = 2.76 \times 10^{-15}$ m  
   b) $p = 2.40 \times 10^{-19}$ kg m/s
3. $p_\alpha = 3.458 \times 10^{-19}$ kg m/s ; $\lambda = 1.92 \times 10^{-15}$ m
4. See diagram.

5. $E_{x-ray} = 7.104 \times 10^{-14}$ J ; Cons of Energy: $E_{x-ray} = E_k + E_{\gamma-ray}'$
   So, $E_{x-ray}' = 2.775 \times 10^{-14}$ J ; $\lambda' = 7.17$ pm
6. a) $\theta = 90^\circ$ $\tilde{p}_{\gamma} \Delta \lambda = 1.3234$ fm ; $\lambda_f = 5.32$ fm ; $f = 5.64 \times 10^{22}$ Hz
   b) Conservation of E: $E_k = 1.24 \times 10^{-11}$ J
   c) Cons of p: $p_{\gamma-ray} = 1.658 \times 10^{-19}$ kg m/s ; $p_{\gamma-ray}' = 1.245 \times 10^{-19}$ kg m/s
   Solving the triangle, $p'_{\gamma'} = 2.07 \times 10^{-19}$ kg m/s ; $\theta = 36.9^\circ$
7. a) $2.29 \times 10^{20}$ Hz  
   b) $1.521 \times 10^{-13}$ J = $9.50625 \times 10^5$ eV = 951 keV
8. $E = 1.12 \times 10^{-13}$ J ; $p_{\text{photon}} = 3.7333 \times 10^{-22}$ kg m/s ; $v = 2.24 \times 10^5$ m/s
9. $72.6^\circ$
10. See diagram.

11. a) $\Delta \lambda = 1.798$ pm ; $\lambda_f = 3.90$ pm
   b) Conservation of Energy: $E_k = 4.37 \times 10^{-14}$ J
   c) $p_{\gamma-ray} = 3.157 \times 10^{-2}$ kg m/s ; $p_{\gamma-ray}' = 1.701 \times 10^{-2}$ kg m/s
   $\tilde{p}_{\gamma} = \tilde{p}_{\gamma}'$  
   $p_{\gamma-ray} = p_{\gamma-ray}' \cos 75^\circ + \tilde{p}_{\gamma x} \tilde{p}_{\gamma x}' = 2.717 \times 10^{-22}$ kg m/s
   $\tilde{p}_{\gamma y} = \tilde{p}_{\gamma y}'$  
   $0 = p_{\gamma-ray}' \sin 75^\circ + \tilde{p}_{\gamma y} \tilde{p}_{\gamma y}' = 1.643 \times 10^{-22}$ kg m/s
   Solving the triangle, $\theta = 31.2^\circ$

12. Photoelectric Effect - the incident photon is fully absorbed by the electron, the electron is emitted, and the electron keeps the rest of the energy as kinetic energy.

Compton Effect - the incident x-ray photon is only partially absorbed, emitting the electron and giving the electron kinetic energy. The rest of the photon energy is reemitted as an x-ray photon with less energy (lower f, longer $\lambda$).
B. MODELS OF THE ATOM

B1. Early Models of the Atom

**Dalton’s Billiard Ball Model** (1810)

- all matter is composed of atoms
- atoms are the smallest particles (quanta) of matter
- there exist no subatomic particles

Advantages: Explained chemical reactions very well
- law of conservation of matter  (the number of atoms remains constant)
- the nature of elements (each element has a unique atom)
- the nature of compounds  (the law of multiple proportions: \( \text{H}_2\text{O} , \text{CO}_2 \))

Disadvantages: It could not explain:
- why molecules form (what holds them together?)
- the repeated properties of elements on the Periodic Table

**Cathode Rays** (Crookes, 1879)

In a cathode ray tube (CRT), two parallel plates were placed within a glass tube and attached to a high voltage source.

When the gas pressure was reduced inside the tube, coloured rays were observed moving from the negative plate (cathode) to the positive plate (anode).

Thus, these rays were called cathode rays.

Were cathode rays particles or waves (EMR)?

Scientists showed conclusively that cathode rays were discrete negative particles, and not EMR.

How?

Cathode rays deflected in magnetic fields and electric fields as a negative charge would.

EMR does not deflect in either field.

Since this obeys LHR #3, cathode rays are negative particles

Only negative particles would attract to the positive plate and repel from the negative plate
**Thomson's Raisin Bun Model** (1897) Also called the "Plum Pudding model"

The discrete negative particles inside cathode rays were called electrons by J.J. Thomson. When electrons were emitted by the cathode, there was no change in mass or deterioration of the cathode metal. This was clear evidence that electrons were particles smaller than atoms, which defied Dalton’s model. Thus, a new model was needed to account for the existence of electrons inside the atom.

Thomson suggested that the electrons were distributed evenly in a positive fluid, much like raisins in a bun.

![Electrons in positive fluid diagram]

- embedded in the positive fluid
- distributed evenly
- very tiny mass

Positive Fluid
- most of the mass and volume
- uniform density
- equal but opposite charge as electrons (if neutral atom)

However, the raisin bun model needed to be tested. This was done by Ernest Rutherford.

**Rutherford's Scattering Experiment** (1911 - 1913)

Rutherford tested the Thomson model of the atom by firing alpha particles through very thin gold foil.

**Prediction:**
Thomson's model predicted that the gold atoms were of **uniform density**.

As a result, there should be no deflection. The alpha particles should go straight through the foil (close to 0°).

**Actual Results:**
- most (99.99%) of the alpha particles went through with little deflection (< 10°)
- however, 1 in 10,000 were deflected by more than 10°
- very rarely, an alpha particle even deflected 180°

Thomson’s model had no explanation for the significant deflections (> 10°).
**Rutherford's Planetary Model** (1913)

Rutherford postulated that most of the atom’s mass is contained in a tiny nucleus, and this nucleus had a very large positive charge. To account for the volume of the atom, Rutherford suggested that the electrons were in orbit around the nucleus.

- **Electrons**
  - in orbit around the positive nucleus

- **Nucleus**
  - tiny, dense, positively-charged
  - contains most of the atomic mass

- **Empty Space**
  - takes up 99.99% of the atomic volume

Rutherford’s planetary model well explained the results of the scattering experiment:

- Most $\alpha$-particles went through with no deflection
  
  Why? The gold atoms are mostly empty space. The $\alpha$-particles move through with no interaction with the gold atoms.

- A few scattered at significant angles ($>10^\circ$)
  
  Why? $\alpha$-particles that get close to the nucleus are deflected by electric repulsion (like charges)
  
  The closer to the nucleus, the greater the deflection.
  
  This was verified using Coulomb’s law.

  Note: If the $\alpha$-particle travelled “head-on”, it would reflect straight backward.

**Major Problem with the Planetary Model**

- if the electron is in orbit around the nucleus, then it is constantly accelerating (centripetally)
- an accelerating charge should emit EMR (Maxwell)
- thus, the electron should constantly be losing energy
- it would spiral in to the nucleus in $10^{-8}$ s!
B2. Properties of the Electron (Charge and mass)

1. Charge-to-Mass Ratio of the Electron  (Thomson, 1897)
Thomson used the modified CRT below to investigate the properties of the electron:

**Step 1:** Thomson turned on both fields so that the cathode rays went through undeflected

Newton's 1st Law: (balanced forces)

\[ |\vec{F}_m| = |\vec{F}_e| \]

\[ qvB_\perp = q|E| \]

\[ vB_\perp = |E| \]

\[ v = \frac{|E|}{B_\perp} \]

Using this approach, Thomson was able to determine the speed of the electrons

**Step 2:** Thomson then turned off the electric field, so that there was only a magnetic field
- the electron would then be deflected into uniform circular motion

\[ |\vec{F}_{net}| = m|\vec{a}| \]

\[ F_m = ma_e \]

\[ qvB_\perp = \frac{mv^2}{r} \]
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Step 3: Thomson then combined the two equations to find the charge-to-mass ratio:

\[
\frac{q}{m} = \frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg}
\]

2. Charge of the Electron (Millikan, 1913)

- Recall, Millikan suspended oil droplets between charged parallel plates

Newton's 1st Law: (balanced forces)

\[
|\vec{F}_e| = |\vec{F}_g|
\]

\[
q |E| = mg
\]

So,

\[
q = \frac{mg}{|E|}
\]  where  \(|E| = \frac{\Delta V}{d}\)

- he discovered two things:
  - the smallest possible charge was \(1.6 \times 10^{-19} \text{ C}\), which he called the elementary charge
  - all other oil-drop charges were multiples of the elementary charge

  i.e.

\[
q = ne
\]

where \(e\) is the elementary charge (\(1.6 \times 10^{-19} \text{ C}\))

\(n\) is the number of electrons in excess or deficit

- Millikan believed that the elementary charge was the charge of an electron (magnitude)

  Charge of an electron = \(e = 1.6 \times 10^{-19} \text{ C} \) (magnitude)

3. Mass of the Electron

- combining the results of Thomson and Millikan, the mass of the electron could be determined

\[
\frac{q}{m} = 1.76 \times 10^{11} \text{ C/kg}
\]

\[
m = \frac{q}{1.76 \times 10^{11} \text{ C/kg}} = \frac{1.6 \times 10^{-19} \text{ C}}{1.76 \times 10^{11} \text{ C/kg}} = 9.1 \times 10^{-31} \text{ kg}
\]

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Unit 5: Atomic Theory

HOMEWORK (Charge-to-mass Ratio)

A. 1. A positively-charged particle travels undeflected through mutually perpendicular electric and magnetic fields, as shown. If the charge is travelling at $6.80 \times 10^5 \text{ m/s}$ and the magnetic field strength is 770 mT, then find the magnitude and direction of the electric field between the plates.

2. An alpha particle is entering a perpendicular magnetic field at a speed of $4.0 \times 10^7 \text{ m/s}$. If the magnetic field is 290 mT acting out of the page, then determine:
   a) the radius of its circular path (in cm)
   b) whether the path is clockwise or counterclockwise

3. A charged particle enters a 75.0 mT magnetic field and goes into circular motion with a diameter of 14.0 cm. If it is moving at a speed of $8.10 \times 10^6 \text{ m/s}$, then find its charge-to-mass ratio.

4. A proton is accelerated from rest through a potential difference of 2.7 kV. What is its final speed?

5. An oil droplet, with a weight of $2.80 \times 10^{-15} \text{ N}$, is suspended between two parallel plates.
   a) What is the charge of the oil drop? Is it positive or negative?
   b) Find the number of electrons in excess or deficit.

6. For the following, sketch the straight-line graph, provide the equation of the line, and determine the physical significance of the slope (including its units):
   Millikan’s oil-drop experiment: Potential difference as a function of charge for suspended oil-drop

B. 7. A negatively-charged particle travels undeflected through mutually perpendicular electric and magnetic fields. The charge is travelling at a speed of $1.80 \times 10^7 \text{ m/s}$. If the voltage between the plates is 500 V and they are 8.00 cm apart, then what is the magnitude and direction of the magnetic field?
8. A particle with a charge-to-mass ratio of $1.33 \times 10^9$ C/kg enters a 70.0 mT perpendicular magnetic field and goes into circular motion with a radius of 16.0 mm. What was its speed?

9. An electron first travels undeflected through a region with mutually perpendicular electric and magnetic fields ($|E| = 7.00 \times 10^4$ V/m and $B_1 = 2.37$ mT). Then, it enters a perpendicular magnetic field ($B_2 = 5.00$ mT) and goes into circular motion. What is the diameter of the circle?

10. A charged particle is accelerated from rest through a potential difference of 950 V and it attains a final speed of $1.80 \times 10^7$ m/s. What is the particle’s charge-to-mass ratio? (Note: Can you show that the units will become C/kg?)

11. An alpha particle is accelerated from rest through a potential difference $V$. When it then enters a perpendicular 760 mT magnetic field, it goes into circular motion with a diameter of 6.00 cm. What is the potential difference $V$? Answer in kV.

12. An oil drop has 7 electrons in excess and it has a mass of $5.40 \times 10^{-16}$ kg. If it is suspended between two parallel plates that are 11.0 cm apart, then what is the required potential difference between the plates?

13. For the following, sketch the straight-line graph, provide the equation of the line, and determine the physical significance of the slope (including its units):

   A charged particle being deflected into circular motion by a perpendicular magnetic field: Radius of circular motion as a function of speed

SOLUTIONS

1. $5.24 \times 10^5$ V/m ; $F_e$ acts up, $F_m$ acts down, and so $\vec{E}$ acts downward ($\downarrow$)

2. a) 2.9 cm  
   b) $F_m$ acts down as it enters the field, so it goes clockwise

3. $1.54 \times 10^9$ C/kg

4. $7.2 \times 10^3$ m/s

5. a) $+4.80 \times 10^{-19}$ C  
   b) 3 electrons in deficit

6. Graph $\Delta V$ vs $\frac{1}{q}$ ; $V = \frac{slope}{q}$ ; Slope is mgd ; Units for slope: V·C, N·m, J, or $\frac{kg \cdot m^2}{s^2}$

7. $3.47 \times 10^{-4}$ T ; $F_e$ acts up, $F_m$ acts down, so $B$ acts into the page ($\times$)

8. $1.49 \times 10^9$ m/s

9. $v = 2.9536 \times 10^7$ m/s ; $r = 3.36$ cm ; diameter = 6.73 cm

10. $1.71 \times 10^{11}$ C/kg

11. $v = 1.0971 \times 10^6$ m/s ; $\Delta V = 1.25 \times 10^4$ V = 12.5 kV

12. 520 V

13. Graph $r$ vs $v$ ; $r = (slope) v$ ; Slope is $\frac{m}{qB_1}$ ; Units for slope: $\frac{kg}{C \cdot T}$ or s
B3. Spectra and the Bohr Model

**Emission and Absorption Spectra**

- There are three types of spectra:

1. Continuous Spectra (dense, heated material)
   - Heated solids, liquids, and dense gases emit light with a **continuous spectrum**
   - The spectrum contains all frequencies (colours), with no gaps
   - EMR is due to the collisions between atoms

2. Emission Spectra
   - Low pressure (rarefied) gases exposed to high voltage give off specific colours (e.g. neon signs)
     - The spectra show a discrete number of **bright bands**

   - Emitted EMR is due to the excited atoms themselves
   - Each element (atom) has a unique emission spectrum (spectral signature)
   - Classical theory could not explain why most frequencies are missing

3. Absorption Spectra
   - When white light passes through unexcited gas, there are dark lines (missing frequencies) in the spectrum

**A gas absorbs the same frequencies of EMR that it emits**
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**Unit 5: Atomic Theory**

**HOMEWORK** (Emission and Absorption Spectra)

1. Identify the type of spectrum associated with each of the following descriptions:
   
   a) low-pressure neon gas is excited by current and gives off light
   b) the light from the sun, after it goes through its (relatively) cold atmosphere
   c) the light emitted from a hot metal
   d) the entire spectrum of colours are present, but there are a few dark lines
   e) all frequencies of EMR are present
   f) there are a few bright lines, but the rest are missing

   *Use the information below to answer the next 2 questions.*

   ![Spectra Diagram]

   The emission spectra for 4 common elements are shown below:

   - Hydrogen (H)
   - Sodium (Na)
   - Helium (He)
   - Mercury (Hg)

2. Determine the identity of the gases in the samples below, based on the spectra:

   ![Sample Spectra]

   a) 
   b) 
   c) 

   **SOLUTIONS**

   1. a) emission  
      b) absorption  
      c) continuous  
      d) absorption  
      e) continuous  
      f) emission

   2. a) He and Na  
      b) H and Hg  
      c) H, He, and Na

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Bohr Model of Hydrogen  (1913)  

- Orbital radii and energies are quantized

Electrons can only be located at certain radii (but not in between). If the electron remains at an allowed orbit, it will not emit EMR despite its acceleration. The lowest possible orbit \((n = 1)\) is lowest energy state (called unexcited or ground state), which is the most stable orbit. All other orbits are called excited states, and they are unstable.

Each allowed orbit (called a stationary state) has a required energy level. The electron must have the exact energy to be there. The energy levels Bohr calculated are shown in the energy diagram:

Note:  When the electron goes beyond \(n = \infty\), it leaves (ionizes) the atom with kinetic energy. The minimum energy to remove an electron from the atom (i.e. to get it to \(n = \infty\)) is called the ionization energy.

- Electrons are in uniform circular motion (orbits)

The electric force of attraction between the proton (nucleus) and the electron is what holds the electron in orbit. That is, the electric force is the centripetal force.

\[
|\vec{F}_{net}| = ma\]  \(F_e = ma_c\)  \(\frac{kq_1q_2}{r^2} = \frac{mv^2}{r}\)  \(\frac{kq^2}{r^2} = \frac{mv^2}{r}\)

- Energy Transitions

- EMR is emitted or absorbed only when the electron jumps orbits (quantum leap)

Based on the law of conservation of energy, the energy gained or lost by the electron (called the transition energy \(\Delta E\)) must be equal to the photon energy.

\[
\Delta E_e = E_{\text{photon}} \quad \text{where} \quad \Delta E_e = E_u - E_L
\]
**Bohr’s Explanation of the Hydrogen Spectra**

**Hydrogen Emission Spectrum**

Why does excited, low pressure hydrogen gas emit only certain frequencies of EMR?

First, the high voltage creates current within the cathode ray tube. The electrons collide with the hydrogen atoms and excite them to higher energy levels.

Since the excited energies level are unstable, the atoms will drop to lower energy levels. When they drop levels, they lose energy and emit photons.

Bohr was able to show that each bright line of the emission spectrum is a unique energy drop.

Note: Absorption Spectrum

Bohr was also able to explain why a gas absorbs the same frequencies of EMR that it emits. To illustrate, Bohr knew that when a hydrogen atom drops from \( n = 4 \) to \( n = 2 \), it emits a green photon with an energy of 2.56 eV (\( \lambda = 486.1 \) nm). Clearly, in order to raise the hydrogen atom from \( n = 2 \) to \( n = 4 \), it requires the same amount of energy (2.56 eV). The only photon capable of providing this exact energy is the same green photon.

The same reasoning applies to every other wavelength emitted (and absorbed) by hydrogen.

**Evaluating Bohr’s Model**

It could explain the emission and absorption spectra for hydrogen (and any other 1-electron atoms).

It could NOT explain - the spectra for more complex atoms (with more than one electron)
- why only certain orbits were allowed
- why the orbiting electrons did not emit EMR, as predicted by Maxwell
- why some spectral lines were brighter than others
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Unit 5: Atomic Theory

HOMEWORK (Bohr’s Model)

1. An electron in Bohr’s H model has a radius is 3.3856 nm. What is the orbital speed?

2. If the orbital speed of the electron in Bohr’s hydrogen model is $2.185 \times 10^5$ m/s, then what is the orbital radius?

For #3 - 9, use the energy diagram for the Bohr hydrogen model shown.

3. An electron in the Bohr hydrogen model drops from $n = 7$ to $n = 3$. What is the wavelength of light emitted? Which type of EMR is this (IR, visible, or UV)?

4. An unexcited hydrogen atom absorbs a photon and it rises to $n = 4$. What is the frequency of the incident photon?

5. An electron in the Bohr hydrogen model drops from $n = 6$ and emits a photon with a wavelength of 2.618 µm. To which lower level did it fall?

6. An unexcited hydrogen atom absorbs a photon with a frequency of $3.154 \times 10^{15}$ Hz. To what level does it rise?

7. An electron is at $n = 5$ in Bohr’s hydrogen model. What is the maximum wavelength of EMR to ionize this atom?

8. Describe and explain what happens when an unexcited hydrogen atom receives low energy incident photons with the following energies.
   (a) 12.1 eV  (b) 11.0 eV  (c) 30 eV

9. Identify the transition in the Bohr hydrogen model associated with the following events.
   Then, using the energy diagram for hydrogen, verify that the proper wavelength results.
   a) emission of violet light (410.1 nm)  b) absorption of red light (656.2 nm)

10. Which type of EMR is emitted for each transition?
   a) $7 \rightarrow 3$  b) $4 \rightarrow 1$  c) $5 \rightarrow 2$  d) $9 \rightarrow 2$

SOLUTIONS

1. $2.73 \times 10^5$ m/s
2. $r = 5.2915 \times 10^{-9}$ m
3. $E_{\text{photon}} = 1.232$ eV; $\lambda = 1.01$ µm (IR, since $\lambda > 700$ nm)
4. $E_{\text{photon}} = 12.75$ eV; $f = 3.08 \times 10^{15}$ Hz
5. $E_{\text{photon}} = 0.4744$ eV; $E_n = -0.8522$ eV; $n_L = 4$
6. $E_{\text{photon}} = 13.058$ eV; $E_n = -0.542$ eV; $n_u = 5$
7. From $n = 5$ to $n = \infty$; $E_{\text{photon}} = \Delta E_e = 0.544$ eV; $\lambda = 2.28$ µm
8. (a) The photon is fully absorbed. It has the exact energy to raise the atom to $n = 3$
   (b) The photon is not absorbed. It does not have the exact energy required.
   (c) The photon is fully absorbed and it ionizes the atom.
   The electron leaves the atom with 16.4 eV of kinetic energy.
9. a) 6 → 2  b) 2 → 3
10. a) Infrared  b) UV  c) Blue  d) UV

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B4. Quantum Mechanics

Matter Waves (de Broglie, 1923)

Compton showed conclusively that high frequency EMR can have momentum and can thus behave like particles. de Broglie suggested, due to a belief in symmetry, that matter could show wavelike properties. This introduced the idea of **wave-particle duality** in particles.

He combined the photon formula \( p = \frac{h}{\lambda} \) with the momentum formula for a particle \( p = mv \)

\[
\lambda = \frac{h}{mv}
\]

**Note:** Works only if \( v \ll c \)

where \( \lambda \) is the wavelength of the matter wave (in m)
\( h \) is Planck’s constant (use \( 6.63 \times 10^{-34} \text{ J-s} \))
\( m \) is the mass of the particle (in kg)
\( v \) is the speed of the particle (in m/s)

**Evidence for matter waves:** (Double Slit Experiment for Electrons)

Prior to quantum theory, there was a clear difference between particle behaviour and wave behaviour. This was certainly evident in the double-slit experiment. If you send large particles through two slits (like firing bullets through slits in a steel plate), then the particles would make a pattern of two vertical lines on the screen, as shown below left. However, if you send a wave through two slits (like sending water waves through two slits in a barrier), then diffraction would take place and the resulting waves would form an interference pattern on the screen, as shown below right. The bright lines would be regions of constructive interference, while the dark lines would be regions of destructive interference. Thus, particles and waves behaved in completely different ways.

![2-slit experiment for large particles](image)

![2-slit experiment for waves](image)

![2-slit experiment for electrons](image)

However, when the same experiment was done on the microscopic scale (i.e. at the quantum level), there was no such clear distinction between particle and wave behaviour.

When scientists fired electrons through two tiny slits, they expected to see a pattern of two vertical lines (much like the diagram above left), since an electron is a particle. However, to their surprise, the electrons formed an interference pattern!

This clearly indicated that electrons had wave properties, which was conclusive evidence for **wave-particle duality** of particles.
Quantum Mechanical Model of the Atom (Schrodinger, 1926)

Scientists were coming to realize that in order to fully understand the atom, they had to embrace the wave-particle duality of the electron. In fact, there was growing evidence that when an electron is in orbit around an atom, it behaves more like a wave than a particle. This proved a problem, since waves have no definite position but are spread over a much larger area. One scientist (Heisenberg) claimed that it was impossible to know the electron’s exact location and momentum at the same time. This was called the Heisenberg Uncertainty Principle.

So, rather than placing electrons in definite orbits (as Rutherford and Bohr did), scientists were forced to consider the location of an electron as a probability function. This is shown below:

![Quantum mechanical model of the hydrogen atom (n = 1)](image)

The region of high probability is called an orbital, and this is where we would most likely find the electron. However, there is a low probability that the electron could be somewhere else. Since we don’t know the actual location of the electron, we cannot describe its path any longer.

Note: These probabilities can be calculated precisely, just like exact predictions at dice or playing cards. These are NOT like the probabilities you would see at sporting events or for natural disasters, which are only estimates.

The quantum mechanical model of the atom has been incredibly successful in predicting atomic behaviour. In fact, its basic structure has not changed since 1930, which is amazing considering all of the technological advances since then. It remains our best mathematical description of the atom to date.

Evaluating the Quantum Mechanical Model

The quantum mechanical model could answer all the questions troubling the Bohr model. It explained:

- The spectra for more complex atoms (with more than one electron)
- Why only certain orbits were allowed (i.e. why orbits were quantized)
  - the electron formed standing waves at these orbits!
- Why the orbiting electrons did not emit EMR
  - since they behave like waves, they no longer behave as accelerating charges
  - thus, they were no longer required to emit EMR
- Why some spectral lines were brighter than others
  - certain transitions had a higher probability of happening, which was outlined clearly by the probability functions in this model
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Unit 5: Atomic Theory

HOMEWORK (Quantum Mechanics)

A. 1. What is the wavelength of:
   a) a 30 g bullet that is moving at a speed of 500 km/h?
   b) an electron that has 25 eV of kinetic energy?

   2. How much energy would a photon have (in MeV) if it had the same wavelength as an
      alpha particle moving at a speed of $7.10 \times 10^5$ m/s?

Both A and B

3. An electron beam passing through a wide slit travel in a straight path. However, if the slit
   width is decreased, then there is much more scattering. Explain the significance of
   this observation.

   ![Diagram of electron beam passing through a wide slit](image)

B. 4. A particle with a charge of $+5e$ and 420 times the mass of an electron is accelerated (from
   rest) through a potential difference $V$. If it attains a wavelength of $2.48 \times 10^{-13}$ m,
   then determine $V$.

   5. When electrons are sent through a double-slit apparatus, the distance between the maxima
   is 3.5 mm. If the distance between the slits is $7.0 \times 10^{-10}$ m and the distance to the screen
   is 6.00 cm, then determine:
      a) the wavelength of the electrons   b) the speed of the electrons.

   6. Electrons are accelerated (from rest) through a potential difference of 4000 V.
      a) What is the wavelength of the electrons? (Ignore relativistic effects)
      b) They are then sent through a double-slit apparatus and the angle to the first-order
         maximum is 11.0°. How far apart must the slits be?

SOLUTIONS

1. a) $1.6 \times 10^{-34}$ m
   b) $E_0 = 4.0 \times 10^{-18}$ J ; $v = 2.963 \times 10^6$ m/s ; $\lambda = 2.5 \times 10^{-10}$ m

2. $\lambda_0 = 1.4042 \times 10^{-13}$ m ; $E_{photon} = 1.4165 \times 10^{-12}$ J = $8.85 \times 10^6$ eV = 8.85 MeV

3. This is consistent with wave theory - as the slit narrows, the amount of diffraction increases.
   This provided further evidence for the wave nature of electrons.

5. $v = 6.987 \times 10^6$ m/s ; $\Delta V = 1.17 \times 10^4$ V

5. a) $4.1 \times 10^{-11}$ m         b) $1.8 \times 10^7$ m/s

6. a) $3.75 \times 10^7$ m/s       b) $\lambda = 1.94 \times 10^{-11}$ m ; $d = 1.02 \times 10^{-10}$ m

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